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THEORETICAL RESULTS FOR STARVED ELLIPTICAL CONTACTS*

It was not until the late 1960's and early 1970's that the influence of lubricant starvation on elastohydrodynamic behavior received serious consideration. Before this time it was assumed that inlets to elastohydrodynamic conjunctions were always fully flooded. This assumption seemed to be entirely reasonable in view of the minute quantities of lubricant required to provide an adequate film. However, in due course, it was recognized that some machine elements suffered from lubricant starvation.

The influence of partial filling of the inlet to an elastohydrodynamic conjunction on pressure and film thickness can readily be explored theoretically by adopting different starting positions for the inlet pressure boundary. Orcutt and Cheng (1965-66) appear to have been the first to proceed in this way for a specific case corresponding to a particular experimental situation. Their results showed that lubricant starvation reduced the film thickness. Wolveridge, et al. (1971) used a Grubin (1949) approach in an analysis of starved elastohydrodynamic lubricated line contacts, and Wedeven, et al. (1971) analyzed a starved condition in a ball-on-plane geometry. Castle

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and Dowson (1972) presented a range of numerical solutions for the starved line-contact elastohydrodynamic situation. In these references the analyses yielded values of the proportional reduction in film thickness from the fully flooded condition in terms of a dimensionless inlet boundary parameter.

In the present chapter, 15 cases in addition to three presented in Chapter 8 were used in a theoretical study of the influence of lubricant starvation on film thickness and pressure in elliptical elastohydrodynamic conjunctions. From the results a simple and important critical dimensionless inlet boundary distance at which lubricant starvation becomes significant was specified. This inlet boundary distance defines whether a fully flooded or a starved condition exists in the contact. Furthermore it was found that the film thickness for a starved condition could be written in dimensionless terms as a function of the inlet distance parameter and the film thickness for a fully flooded condition. Contour plots of pressure and film thickness in and around the contact are shown for fully flooded and starved conditions. The theoretical findings are compared directly with results reported previously by Wedeven, et al. (1971). This chapter also makes extensive use of the work presented by Hamrock and Dowson (1977b).

9.1 Fully Flooded - Starved Boundary

The computing area in and around the Hertzian contact is shown in Figure 9.1. In this figure the coordinate X is made dimensionless with respect to the semiminor axis b of the contact ellipse, and the coordinate Y is made dimensionless with respect to the semimajor axis a of the contact ellipse as defined in Chapter 7. The ellipticity parameter k is defined as the semimajor axis divided by the semiminor axis of the contact ellipse ($k = a/b$). Because of the dimensionless form of the coordinates X and Y the Hertzian contact ellipse becomes a Hertzian circle regardless of the value of k. This Hertzian contact circle is shown in Figure 9.1 with a radius of unity. The edges of the computing area, where the pressure is assumed to be ambient, are also denoted. In this figure the dimensionless inlet distance $\tilde{m}$, which is equal to the dimensionless distance from the center of the Hertzian contact zone to the inlet edge of the computing area, is also shown.

Lubricant starvation can be studied by simply changing the dimensionless inlet distance $\tilde{m}$. A fully flooded condition is said to exist when the dimensionless inlet distance ceases to influence the minimum film thickness to any significant extent.

The location of the dimensionless inlet distance at which the minimum film thickness first starts to change when $\tilde{m}$ is gradually reduced from a fully flooded condition is called the

fully flooded - starved boundary position and is denoted by m^*. Therefore lubricant starvation was stuuied oy using the basic elas ydrodynamic lubrication elliptical-contact theory developed in Chapter 7 and by observing how a reduction in the dimensionless inlet distance affected the basic features of the conjunction.

The influence of changes in the dimensionless inlet distance on the dimensionless minimum film thickness is shown in Table 9.1 for three groups of dimensionless load and speed parameters. For all the results presented in this chapter the materials parameter G was fixed at 4522 and the ellipticity parameter, at 6. It can be seen from Table 9.1 that, as the dimensionless inlet distance $\tilde{m}$ decreases, the dimensionless minimum film thickness H_{min} also decreases.

The influence of the three groups of dimensionless speed and load parameters considered on the location of the dimensionless inlet boundary distance m^* is shown in Table 9.2. Also given in this table are the corresponding values of the dimensionless central and minimum film thicknesses for the fully flooded condition as obtained by interpolating the numerical values. The value of the dimensionless inlet boundary position m^* shown in Table 9.2 was obtained by using the data from Table 9.1 when the following equation was satisfied:

$$\frac{H_{min} - (H_{min})_{\tilde{m}=m^*}}{H_{min}} = 0.03 \qquad (9.1)$$

The value of 0.03 was used in equation (9.1) since it was ascertained that the data in Table 9.1 were accurate to only ±3 percent.

The general form of the equation that relates the dimensionless inlet distance at the fully flooded - starved boundary m^* to the geometry and central film thickness of an elliptical elastohydrodynamic conjunction can be written as

$$m^* - 1 = A\left[\left(\frac{R_x}{b}\right)^2 H_c\right]^B \tag{9.2}$$

The right side of equation (9.2) is similar in form to the equations given by Wolveridge, et al. (1971) and Wedeven, et al. (1971). By applying a least-squares power fit to the data obtained from Table 9.1 we can write

$$m^* = 1 + 3.06\left[\left(\frac{R_x}{b}\right)^2 H_c\right]^{0.58} \tag{9.3}$$

A fully flooded condition exists when $\tilde{m} \geq m^*$, and a starved condition exists when $\tilde{m} < m^*$. The coefficient of determination tion ε for these results is 0.9902, which is entirely satisfactory.

If the dimensionless minimum film thickness is used in equation (9.2) instead of the central film thickness, we obtain

$$m^* = 1 + 3.34\left[\left(\frac{R_x}{b}\right)^2 H_{min}\right]^{0.56} \tag{9.4}$$

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The coefficient of determination for these results is 0.9869, which is again very good.

When Wedeven's expression for the dimensionless inlet distance at the fully flooded - starved boundary is rewritten in terms of the variables considered in this text and by Hamrock and Dowson (1977b), it becomes

$$m_W = 1 + 3.52\left[\left(\frac{R_x}{b}\right)^2 H_c\right]^{2/3} \qquad (9.5)$$

It is evident that there is close and encouraging agreement between equations (9.3) and (9.5). The latter, however, predicts a slightly higher value for the location of the fully flooded - starved boundary than is predicted from the present results.

9.2 Equations for Film Thickness in Starved Elastohydrodynamic Elliptical Conjunctions

Having clearly established the limiting location of the inlet boundary required to ensure fully flooded conditions, equations (9.3) and (9.4), we can develop an equation defining the dimensionless film thickness for elliptical conjunctions operating under starved lubrication conditions. The ratio between the dimensionless central film thickness in starved and fully flooded conditions can be expressed in general form as

$$\frac{H_{c,s}}{H_c} = C\left(\frac{\tilde{m} - 1}{m^* - 1}\right)^D \qquad (9.6)$$

The influence of the ratio of the dimensionless inlet distance parameter to the dimensionless distance to the fully flooded - starved boundary $(\tilde{m} - 1)/(m^* - 1)$ on the ratio of central film thickness in the starved and fully flooded conditions $H_{c,s}/H_c$ is shown in Table 9.3. A least-squares power curve fit to the 16 pairs of data points

$$\left[\left(\frac{H_{c,s}}{H_c}\right)_i, \left(\frac{\tilde{m} - 1}{m^* - 1}\right)_i\right], \qquad i = 1,2, \ldots, 16$$

was used in obtaining values for C and D in equation (9.6). For these values of C and D the dimensionless central film thickness for a starved lubrication condition can be written as

$$H_{c,s} = H_c \left(\frac{\tilde{m} - 1}{m^* - 1}\right)^{0.29} \tag{9.7}$$

By using a similar approach and the data in Table 9.3 the dimensionless minimum film thickness for a starved lubrication condition can be written as

$$H_{min,s} = H_{min} \left(\frac{\tilde{m} - 1}{m^* - 1}\right)^{0.25} \tag{9.8}$$

Therefore, whenever $\tilde{m} < m^*$, where m^* is defined by either equation (9.3) or (9.4), a starved lubrication conditions exists. When this is true, the dimensionless central film thickness is expressed by equation (9.7), and the dimensionless minimum film thickness is expressed by equation (9.8). If $\tilde{m} \geq m^*$, where m^* is defined by either equation (9.3) or (9.4),

a fully flooded condition exists. Expressions for the dimensionless central and minimum film thicknesses for a fully flooded condition (H_c and H_{min}) were developed in Chapter 8 and are expressed in equations (8.41) and (8.23), respectively.

The ratio of the dimensionless inlet distance to the dimensionless location of the fully flooded - starved boundary as obtained from Wedeven, et al. (1971), expressed as $(\tilde{m} - 1)/(m_W - 1)$, is also given in Table 9.3. By comparing these results with those obtained by Hamrock and Dowson (1977b), $(\tilde{m} - 1)/(m^* - 1)$, it can be seen that for group 1 the agreement is excellent. However, for groups 2 and 3 the agreement is not so good. A possible explanation for this difference is that an approximate expression was used for the Hertzian deformation by Wedeven, et al. (1971). They indicated that their equation, reproduced here as equation (9.5), was valid only for small values of m^*, or more specifically, $m^* < 3$. In group 2, $m^* = 3.71$ and in group 3, $m^* = 5.57$. Since no such assumption was necessary in the derivation of equations (9.3) and (9.4), they would appear to be more general.

The influence of the dimensionless inlet boundary location parameter on central film thickness is shown in Figure 9.2 for the Wedeven, et al. (1971) and Hamrock and Dowson (1977b) results. It can be seen that the Wedeven, et al. (1971) results predict slightly higher values of the central film thickness

under starved conditions than the Hamrock and Dowson (1977b) results.

9.3 Contour Plots of Results

To explain more fully what happens as the degree of lubricant starvation increases, a number of contour diagarams for pressure and film thickness are presented in Figures 9.3 to 9.8. As in Chapter 8 the + symbol indicates the center of the Hertzian contact, and the asterisks indicate the Hertzian contact circle. The contours on each figure are labeled, and tables showing the corresponding values of the dimensionless pressure or film thickness are given in each figure.

In Figures 9.3(a), (b), and (c) contour plots of dimensionless pressure ($P = p/E'$) are given for the conditions represented by the data recorded as group 1 of Table 9.2 and for dimensionless inlet distances $\tilde{m}$ of 4, 2, and 1.25, respectively. In these figures the contour values are the same in each plot. The pressure spikes are evident in Figures 9.3(a) and (b), but not in Figure 9.3(c). This implies that as the dimensionless inlet distance $\tilde{m}$ decreases, or as the severity of lubricant starvation increases, the pressure spike is suppressed. Figure 9.3(a), with $\tilde{m} = 4$, corresponds to a fully flooded condition; Figure 9.3(b), with $\tilde{m} = 2$, to a starved condition; and Figure 9.3(c), with $\tilde{m} = 1.25$, to a severely starved condition. Once

lubricant starvation occurs, the severity of the conditions within the conjunction increases rapidly as $\tilde{m}$ is decreased and dry contact conditions are approached.

Contour plots of the dimensionless film thickness ($H = h/R_x$) for the data presented by results shown in group 1 of Table 9.2 and for conditions corresponding to the three pressure distributions shown in Figure 9.3 are reproduced in Figure 9.4. It is clear that the film shape in the central region of the elastohydrodynamic conjunction becomes more parallel as lubricant starvation increases and that the region occupied by the minimum film thickness becomes more concentrated. Note also that the values attached to the film thickness contours for the starved condition (Figure 9.4(c)) are much smaller than those of the film thickness contours for the fully flooded condition (Figure 9.4(a)).

In Figures 9.5(a), (b), and (c) contour plots of dimensionless pressure ($P = p/E'$) are given for the results shown in group 3 of Table 9.2 and $\tilde{m}$ of 6, 2.5, and 1.5, respectively. The contour values are the same in each plot. Figure 9.5(a) depicts a fully flooded condition; and Figure 9.5(c), a severely starved condition. The following observations can be made about Figure 9.5:

(1) The distance from the center of the contact to the upstream location of the largest contour (labeled H) decreases as the severity of lubricant starvation increases.

(2) Contours A, B, and C, which represent pressures in the region of a pressure spike in fully flooded and partially starved conditions, are absent in the severely starved condition shown in Figure 9.5(c) since the pressure in the latter case is closer to the Hertzian distribution.

In Figures 9.6(a), (b), and (c) contour plots of the dimensionless film thickness ($H = h/R_x$) are shown for the same set of results and the same dimensionless inlet distances $\tilde{m}$ as in Figure 9.5. In the fully flooded condition (Figure 9.6(a)) the minimum film thickness is located to the sides of the conjunction in two areas that are midway between the center of the contact and the Hertzian ellipse. In the severely starved condition shown in Figure 9.6(c) the central portion has roughly parallel contours in the direction of motion, with one minimum-film-thickness area directly behind the axial center of the contact and near the edge of the Hertzian ellipse. Ranger, et al. (1975) found a similar distribution of contours. Note the similarity between the film thickness contours of Figure 9.4(c) (group 1 of Table 9.2) and those of Figure 9.6(c) (group 3 of Table 9.2). It can be seen from the labels of the contours in Figure 9.6 that the film thicknesses for the starved condition are much lower than the film thicknesses for the fully flooded condition.

In Figures 9.7(a) and (b) the dimensionless pressure ($P = p/E'$) distribution along the X axis is shown for three val-

ues of $\tilde{m}$ and for conditions represented by the data in groups 1 and 3 of Table 9.2, respectively. The value of Y was held constant at a value representative of conditions close to the axis of symmetry of the conjunction for these calculations. The pressure spike diminishes as the severity of starvation increases and dry contact conditions are approached.

In Figures 9.8(a) and (b) the dimensionless film thickness ($H = h/R_x$) on the X axis is shown for three values of $\tilde{m}$ and for conditions represented by the data for groups 1 and 3 of Table 9.2, respectively. Once again the value of Y was held fixed and close to the axis of symmetry of the contact in these calculations. It is clear, particularly in Figure 9.8, that the central region becomes flatter as starvation develops. Also in going from a fully flooded condition to a starved condition the film thickness decreases substantially.

9.4 Inlet Boundary Condition

The results presented thus far in this chapter demonstrate the influence of lubricant starvation on the performance of elastohydrodynamic conjunctions. This situation may be encountered in a number of ball bearing applications, particularly where the lubricant supply is restricted. For example, in precision gyroscopes used for inertial navigation the miniature ball bearings are often initially charged with a small amount of

lubricant within a porous separator or cage. In such cases the minute reservoir of lubricant available at the ball-race conjunction is most unlikely to represent a fully flooded situation.

Even in less demanding ball bearing applications, lubricant starvation can represent a severe problem. The lubricant is generally supplied infrequently as oil or grease, and in the latter case the bearing has to operate for long periods of time without being recharged. Even when oil is supplied as a jet or a mist, the passage of a ball over the race sweeps aside the main layer of lubricant to leave a thin film on the surface that may recover only partially before the arrival of the next ball. The question of lubricant film recovery on the races of bearings has been examined theoretically by Chiu (1974).

The major problem with assessing the influence of lubricant starvation on film thickness in elastohydrodynamic conjunctions is that the degree of starvation is rarely known with any accuracy in practical situations. The correction factors presented in this chapter can readily be introduced, provided the location of the inlet meniscus is known in relation to the scale of the Hertzian contact zone. Such information can often be derived directly from laboratory studies of elastohydrodynamic contacts involving the use of transparent components and interferometry, but practical bearing situations are more difficult to deal with.

Nevertheless a useful approach has been developed by Dowson, et al. (1979) in relation to nominal line or rectangular

conjunctions. There is now considerable evidence to support the view that an elastohydrodynamic conjunction initially charged with a fixed amount of lubricant gradually adjusts itself until the inlet meniscus sits at a location at which reverse flow just ceases. Castle and Dowson (1972) and Dowson (1975) have developed this zero-reverse-flow inlet boundary condition and confirmed its merit in disc machine experiments (Dowson, et al., 1979).

The zero-reverse-flow inlet boundary condition requires the lubricant-air meniscus at the entry to the lubricated conjunction to be located at a position where the velocity distribution at some location in the film satisfies the requirement that

$$u = \frac{du}{dz} = 0$$

The ratio of the inlet film thickness h_i to the film thickness at the point of maximum pressure h_m, where $dp/dx = 0$, is then given by

$$\frac{h_i}{h_m} = \frac{3}{2 \pm (1 - \bar{u}^2)^{1/2}} \qquad (9.9)$$

where

$$\bar{u} = \frac{u_a - u_b}{u_a + u_b}$$

and u_a and u_b represent the velocities of the bearing surfaces. In pure rolling $\bar{u} = 0$ and, if one of the solids is stationary while the other moves with velocity u, $\bar{u} = 1$. For these conditions h_i/h_m adopts the values 3 and 1.5, respec-

tively. These film thickness ratios enable the location of the inlet meniscus to be determined for either rigid or elastohydrodynamic conditions.

For rigid cylinders in line contact a degree of lubricant starvation consistent with the zero-reverse-flow boundary condition causes the minimum film thickness to be reduced to about 48 percent of the fully flooded value in pure rolling. For the elastohydrodynamic lubrication of cylinders the corresponding minimum-film-thickness reduction factor is about 71 percent.

The approach is particularly useful when the film thickness is to be calculated in machine elements in which the conjunctions are starved but the degree of starvation is unknown. The zero-reverse-flow inlet boundary condition has a sound physical basis, and there is evidence to suggest that a conjunction initially charged with lubricant will gradually adjust itself in terms of lubricant disposition until the condition is satisfied. Film thicknesses calculated on this basis thus represent conservative estimates of the real situation.

9.5 Closure

The theory and numerical procedures outlined in Chapter 7 have been used to investigate the influence of lubricant starvation on minimum film thickness in starved elliptical elastohydrodynamic conjunctions. This study of lubricant starvation was

performed by moving the inlet boundary closer to the center of the conjunction and calculating the resulting pressure distributions and film shapes. The re ts show that the critical location of the dimensionless inlet boundary which forms a demarcation between fully flooded and starved conditions m^* can be expressed simply as

$$m^* = 1 + 3.06\left[\left(\frac{R_x}{b}\right)^2 H_c\right]^{0.58}$$

or

$$m^* = 1 + 3.34\left[\left(\frac{R_x}{b}\right)^2 H_{min}\right]^{0.56}$$

That is, for a dimensionless inlet distance $\tilde{m}$ less than m^*, starvation occurs and, for $\tilde{m} \geq m^*$, a fully flooded condition exists. Furthermore it has been possible to express the central and minimum film thicknesses for a starved lubrication condition as

$$H_{c,s} = H_c\left(\frac{\tilde{m} - 1}{m^* - 1}\right)^{0.29}$$

$$H_{min,s} = H_{min}\left(\frac{\tilde{m} - 1}{m^* - 1}\right)^{0.25}$$

Contour diagrams for the pressure and film thickness in and around the contact have been presented for both fully flooded and starved conditions. It is evident from the contour diagrams that, as the severity of starvation increases, the pressure

spike becomes suppressed, the film shape becomes more nearly parallel over a substantial part of the Hertzian contact ellipse, and the film thickness decreases substantially.

Attention has been drawn to the significance of a particular inlet boundary condition corresponding to a solution of zero reverse flow. Studies of line contacts have indicated that many elastohydrodynamic conjunctions initially charged with lubricant or provided with restricted quantities of fluid operate close to this unique boundary condition.

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SYMBOLS

A	constant used in equation (3.113)
A^*, B^*, C^*, D^*, L^*, M^*	relaxation coefficients
A_v	drag area of ball, m^2
a	semimajor axis of contact ellipse, m
$\bar{a}$	$a/2\bar{m}$
B	total conformity of bearing
b	semiminor axis of contact ellipse, m
$\bar{b}$	$b/2\bar{m}$
C	dynamic load capacity, N
C_v	drag coefficient
$C_1,\ldots,C_8$	constants
c	19,609 N/cm^2 (28,440 lbf/in^2)
$\bar{c}$	number of equal divisions of semimajor axis
D	distance between race curvature centers, m
$\tilde{D}$	material factor
$\bar{D}$	defined by equation (5.63)
De	Deborah number
d	ball diameter, m
$\bar{d}$	number of divisions in semiminor axis
d_a	overall diameter of bearing (Figure 2.13), m
d_b	bore diameter, m
d_e	pitch diameter, m
d'_e	pitch diameter after dynamic effects have acted on ball, m
d_i	inner-race diameter, m
d_o	outer-race diameter, m

E	modulus of elasticity, N/m^2
E'	effective elastic modulus, $2\Big/\left(\frac{1-\nu_a^2}{E_a}+\frac{1-\nu_b^2}{E_b}\right)$, N/m^2
E_a	internal energy, m^2/s^2
$\tilde{E}$	processing factor
E_1	$[(\tilde{H}_{min} - H_{min})/H_{min}] \times 100$
$\mathcal{E}$	elliptic integral of second kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathcal{E}}$	approximate elliptic integral of second kind
e	dispersion exponent
F	normal applied load, N
F^*	normal applied load per unit length, N/m
$\tilde{F}$	lubrication factor
$\bar{F}$	integrated normal applied load, N
F_c	centrifugal force, N
F_{max}	maximum normal applied load (at $\psi = 0$), N
F_r	applied radial load, N
F_t	applied thrust load, N
F_ψ	normal applied load at angle ψ, N
$\mathcal{F}$	elliptic integral of first kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathcal{F}}$	approximate elliptic integral of first kind
f	race conformity ratio
f_b	rms surface finish of ball, m
f_r	rms surface finish of race, m
G	dimensionless materials parameter, αE
G^*	fluid shear modulus, N/m^2
$\tilde{G}$	hardness factor
g	gravitational constant, m/s^2

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g_E	dimensionless elasticity parameter, $W^{8/3}/U^2$
g_V	dimensionless viscosity parameter, GW^3/U^2
H	dimensionless film thickness, h/R_x
$\hat{H}$	dimensionless film thickness, $H(W/U)^2 = F^2h/u^2\eta_0^2R_x^3$
H_c	dimensionless central film thickness, h_c/R_x
$H_{c,s}$	dimensionless central film thickness for starved lubrication condition
H_f	frictional heat, N m/s
H_{min}	dimensionless minimum film thickness obtained from EHL elliptical-contact theory
$H_{min,r}$	dimensionless minimum film thickness for a rectangular contact
$H_{min,s}$	dimensionless minimum film thickness for starved lubrication condition
$\tilde{H}_c$	dimensionless central film thickness obtained from least-squares fit of data
$\tilde{H}_{min}$	dimensionless minimum film thickness obtained from least-squares fit of data
$\bar{H}_c$	dimensionless central-film-thickness - speed parameter, $H_cU^{-0.5}$
$\bar{H}_{min}$	dimensionless minimum-film-thickness - speed parameter, $H_{min}U^{-0.5}$
$\bar{H}_0$	new estimate of constant in film thickness equation
h	film thickness, m
h_c	central film thickness, m
h_i	inlet film thickness, m

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h_m	film thickness at point of maximum pressure, where $dp/dx = 0$, m
h_{min}	minimum film thickness, m
h_0	constant, m
I_d	diametral interference, m
I_p	ball mass moment of inertia, m N s^2
I_r	integral defined by equation (3.76)
I_t	integral defined by equation (3.75)
J	function of k defined by equation (3.8)
J^*	mechanical equivalent of heat
$\bar{J}$	polar moment of inertia, m N s^2
K	load-deflection constant
k	ellipticity parameter, a/b
$\bar{k}$	approximate ellipticity parameter
$\tilde{k}$	thermal conductivity, N/s °C
k_f	lubricant thermal conductivity, N/s °C
L	fatigue life
L_a	adjusted fatigue life
L_t	reduced hydrodynamic lift, from equation (6.21)
$L_1,\ldots,L_4$	lengths defined in Figure 3.11, m
L_{10}	fatigue life where 90 percent of bearing population will endure
L_{50}	fatigue life where 50 percent of bearing population will endure
ℓ	bearing length, m
$\bar{\ell}$	constant used to determine width of side-leakage region
M	moment, Nm

M_g	gyroscopic moment, Nm
M_p	dimensionless load-speed parameter, $WU^{-0.75}$
M_s	torque required to produce spin, N m
m	mass of ball, N s^2/m
m^*	dimensionless inlet distance at boundary between fully flooded and starved conditions
$\tilde{m}$	dimensionless inlet distance (Figures 7.1 and 9.1)
$\bar{m}$	number of divisions of semimajor or semiminor axis
m_W	dimensionless inlet distance boundary as obtained from Wedeven, et al. (1971)
N	rotational speed, rpm
n	number of balls
n^*	refractive index
$\bar{n}$	constant used to determine length of outlet region
P	dimensionless pressure
P_D	dimensionless pressure difference
P_d	diametral clearance, m
P_e	free endplay, m
P_{Hz}	dimensionless Hertzian pressure, N/m^2
p	pressure, N/m^2
p_{max}	maximum pressure within contact, $3F/2\pi ab$, N/m^2
$p_{iv,as}$	isoviscous asymptotic pressure, N/m^2
Q	solution to homogeneous Reynolds equation
Q_m	thermal loading parameter
$\bar{Q}$	dimensionless mass flow rate per unit width, $q\eta_0/\rho_0 E'R^2$
q_f	reduced pressure parameter
q_x	volume flow rate per unit width in x direction, m^2/s

q_y	volume flow rate per unit width in y direction, m^2/s
R	curvature sum, m
R_a	arithmetical mean deviation defined in equation (4.1), m
R_c	operational hardness of bearing material
R_x	effective radius in x direction, m
R_y	effective radius in y direction, m
r	race curvature radius, m
r_{ax}, r_{bx}, r_{ay}, r_{by}	radii of curvature, m
r_c, ϕ_c, z	cylindrical polar coordinates
r_s, θ_s, ϕ_s	spherical polar coordinates
$\bar{r}$	defined in Figure 5.4
S	geometric separation, m
S^*	geometric separation for line contact, m
S_0	empirical constant
s	shoulder height, m
T	τ_0/p_{max}
$\tilde{T}$	tangential (traction) force, N
T_m	temperature, °C
T_b^*	ball surface temperature, °C
T_f^*	average lubricant temperature, °C
ΔT^*	ball surface temperature rise, °C
T_1	$(\tau_0/p_{max})_{k=1}$
T_v	viscous drag force, N
t	time, s
t_a	auxiliary parameter
u_B	velocity of ball-race contact, m/s

u_c	velocity of ball center, m/s
U	dimensionless speed parameter, $\eta_0 u/E'R_x$
u	surface velocity in direction of motion, $(u_a + u_b)/2$, m/s
$\overline{u}$	number of stress cycles per revolution
Δu	sliding velocity, $u_a - u_b$, m/s
v	surface velocity in transverse direction, m/s
W	dimensionless load parameter, $F/E'R^2$
w	surface velocity in direction of film, m/s
X	dimensionless coordinate, x/R_x
Y	dimensionless coordinate, y/R_x
X_t, Y_t	dimensionless grouping from equation (6.14)
X_a, Y_a, Z_a	external forces, N
Z	constant defined by equation (3.48)
Z_1	viscosity pressure index, a dimensionless constant
$x, \tilde{x}, \overline{x}, \overline{x}_1$; $y, \tilde{y}, \overline{y}, \overline{y}_1$; $z, \tilde{z}, \overline{z}, \overline{z}_1$	coordinate system
α	pressure-viscosity coefficient of lubrication, m^2/N
α_a	radius ratio, R_y/R_x
β	contact angle, rad
β_f	free or initial contact angle, rad
β'	iterated value of contact angle, rad
Γ	curvature difference
γ	viscous dissipation, N/m^2 s
$\dot{\gamma}$	total strain rate, s^{-1}
$\dot{\gamma}_e$	elastic strain rate, s^{-1}
$\dot{\gamma}_v$	viscous strain rate, s^{-1}

γ_a	flow angle, deg
δ	total elastic deformation, m
δ^*	lubricant viscosity temperature coefficient, $^{o}C^{-1}$
δ_D	elastic deformation due to pressure difference, m
δ_r	radial displacement, m
δ_t	axial displacement, m
δ_x	displacement at some location x, m
$\bar{\delta}$	approximate elastic deformation, m
$\tilde{\delta}$	elastic deformation of rectangular area, m
ϵ	coefficient of determination
ϵ_1	strain in axial direction
ϵ_2	strain in transverse direction
ζ	angle between ball rotational axis and bearing centerline (Figure 3.10)
ζ_a	probability of survival
η	absolute viscosity at gauge pressure, N s/m^2
$\bar{\eta}$	dimensionless viscosity, η/η_0
η_0	viscosity at atmospheric pressure, N s/m^2
η_∞	6.31×10^{-5} N s/m^2(0.0631 cP)
θ	angle used to define shoulder height
Λ	film parameter (ratio of film thickness to composite surface roughness)
λ	equals 1 for outer-race control and 0 for inner-race control
λ_a	second coefficient of viscosity
λ_b	Archard-Cowking side-leakage factor, $(1 + 2/3\ \alpha_a)^{-1}$
λ_c	relaxation factor

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μ	coefficient of sliding friction
μ^*	$\bar{\rho}/\bar{\eta}$
ν	Poisson's ratio
ξ	divergence of velocity vector, $(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z)$, s^{-1}
ρ	lubricant density, N s^2/m^4
$\bar{\rho}$	dimensionless density, ρ/ρ_0
ρ_0	density at atmospheric pressure, N s^2/m^4
σ	normal stress, N/m^2
σ_1	stress in axial direction, N/m^2
τ	shear stress, N/m^2
τ_0	maximum subsurface shear stress, N/m^2
$\tilde{\tau}$	shear stress, N/m^2
$\tilde{\tau}_e$	equivalent stress, N/m^2
$\tilde{\tau}_L$	limiting shear stress, N/m^2
Φ	ratio of depth of maximum shear stress to semiminor axis of contact ellipse
Φ^*	$P_H^{3/2}$
Φ_1	$(\Phi)_{k=1}$
ϕ	auxiliary angle
ϕ_T	thermal reduction factor
ψ	angular location
ψ_ℓ	limiting value of ψ
Ω_i	absolute angular velocity of inner race, rad/s
Ω_o	absolute angular velocity of outer race, rad/s
ω	angular velocity, rad/s
ω_B	angular velocity of ball-race contact, rad/s
ω_b	angular velocity of ball about its own center, rad/s

ω_c	angular velocity of ball around shaft center, rad/s
ω_s	ball spin rotational velocity, rad/s
Subscripts:	
a	solid a
b	solid b
c	central
bc	ball center
IE	isoviscous-elastic regime
IR	isoviscous-rigid regime
i	inner race
K	Kapitza
min	minimum
n	iteration
o	outer race
PVE	piezoviscous-elastic regime
PVR	piezoviscous-rigid regime
r	for rectangular area
s	for starved conditions
x,y,z	coordinate system
Superscript:	
$(\bar{\ })$	approximate

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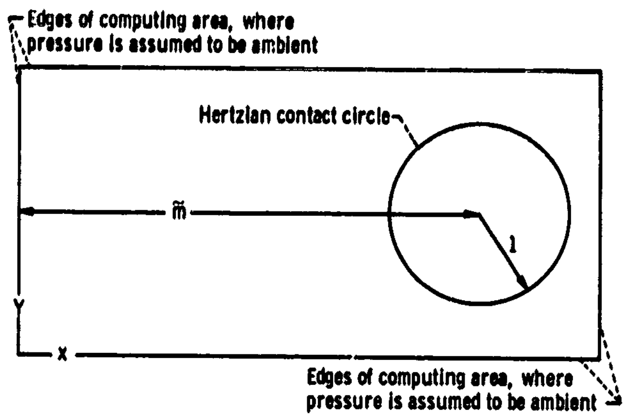


Figure 9.1. - Computing area in and around Hertzian contact zone.

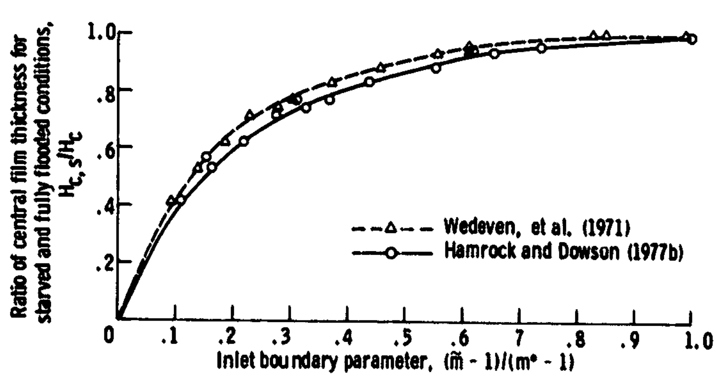
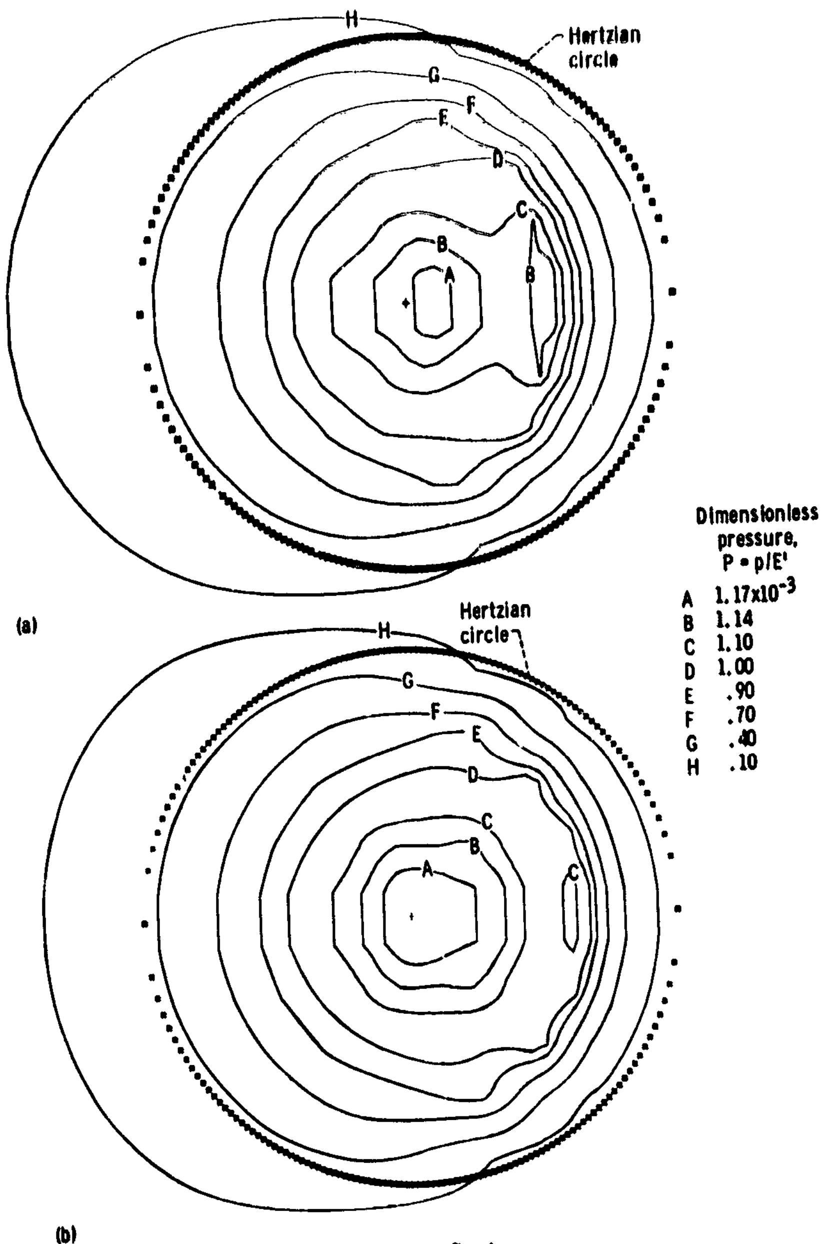
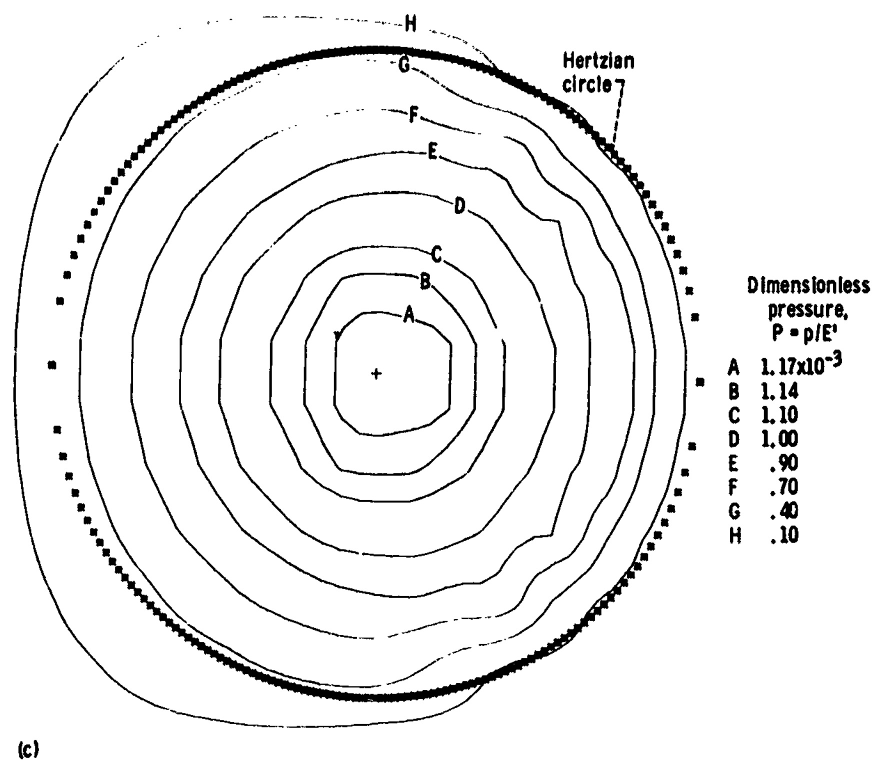


Figure 9.2. - Influence of inlet boundary location parameter on central-film-thickness ratio.



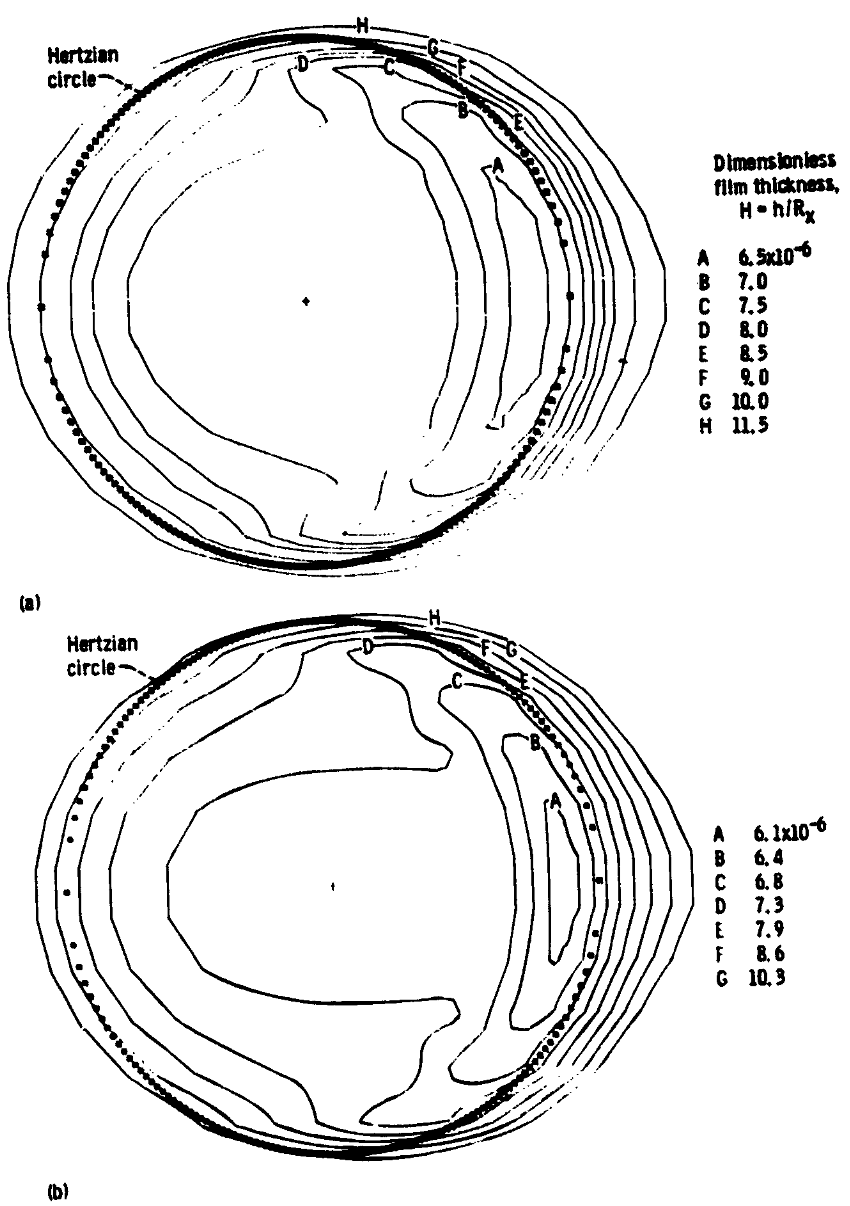
(a) $\bar{m}$ = 4.
(b) $\bar{m}$ = 2.

Figure 9.3. - Contour plots of dimensionless pressure for dimensionless inlet distances $\bar{m}$ of 4, 2, and 1.25 and for group 1 of Table 9.2.



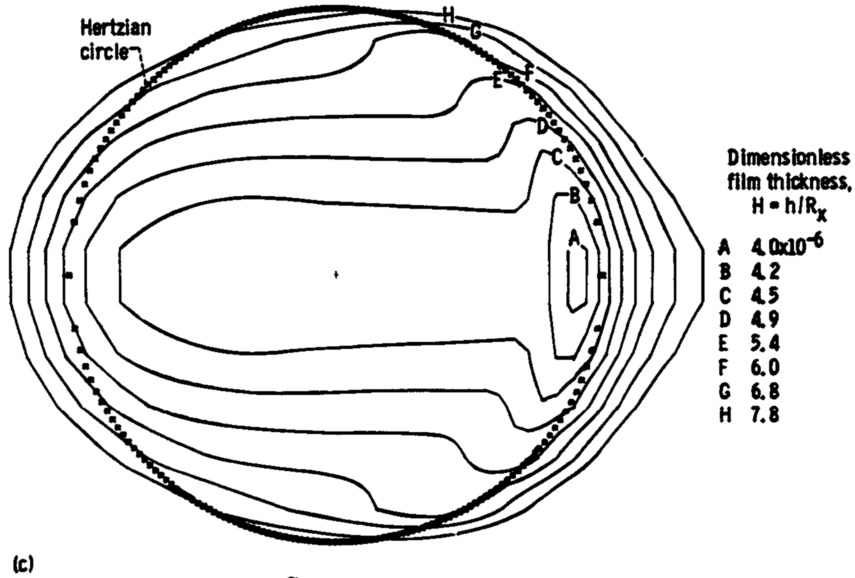
(c) $\tilde{m}$ = 1.25.

Figure 9.3. - Concluded.



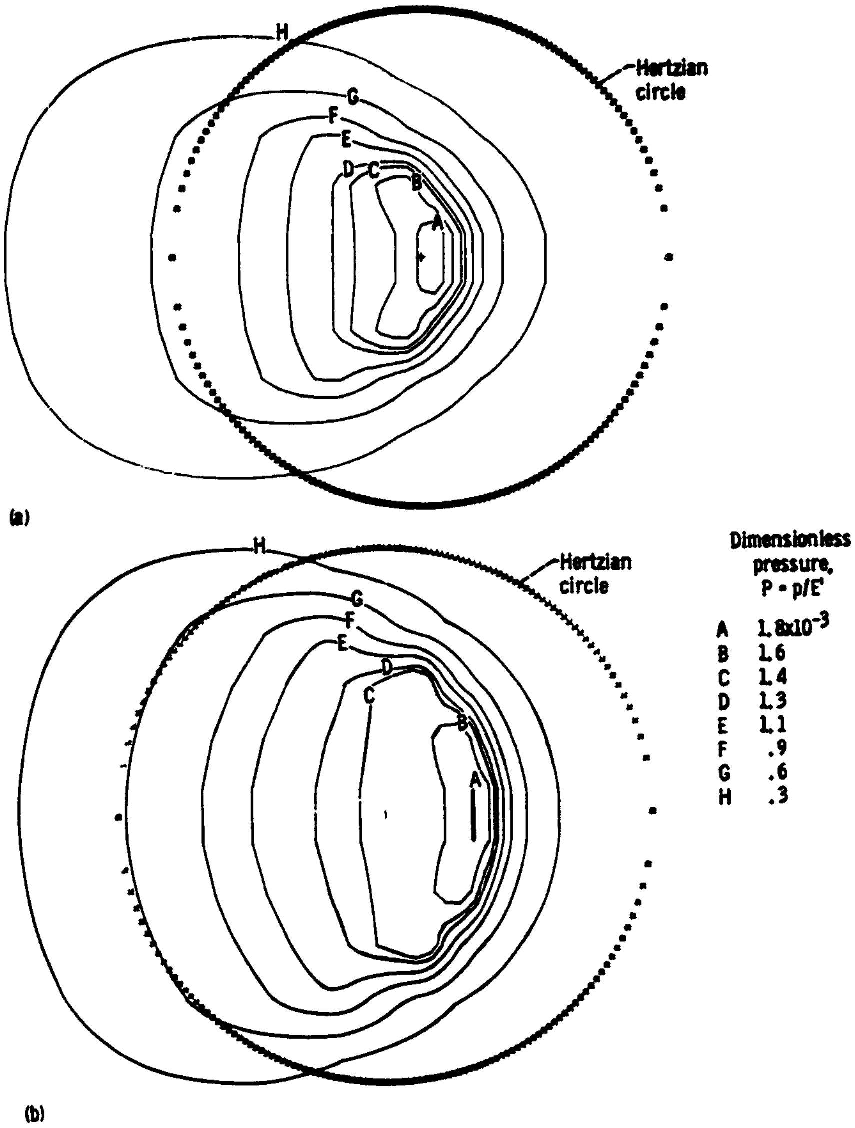
(a) $\bar{m} = 4$.
(b) $\bar{m} = 2$.

Figure 9.4. - Contour plots of dimensionless film thickness for dimensionless inlet distances $\bar{m}$ of 4, 2, and 1.25 and for group 1 of Table 9.2.



(c) $\bar{m} = 1.25$.

Figure 9.4. - Concluded.



(a) $\bar{m} = 6$.
(b) $\bar{m} = 2.5$.

Figure 9.5. - Contour plots of dimensionless pressure for dimensionless inlet distances $\bar{m}$ of 6, 2.5, and 1.5 and for group 3 of Table 9.2.

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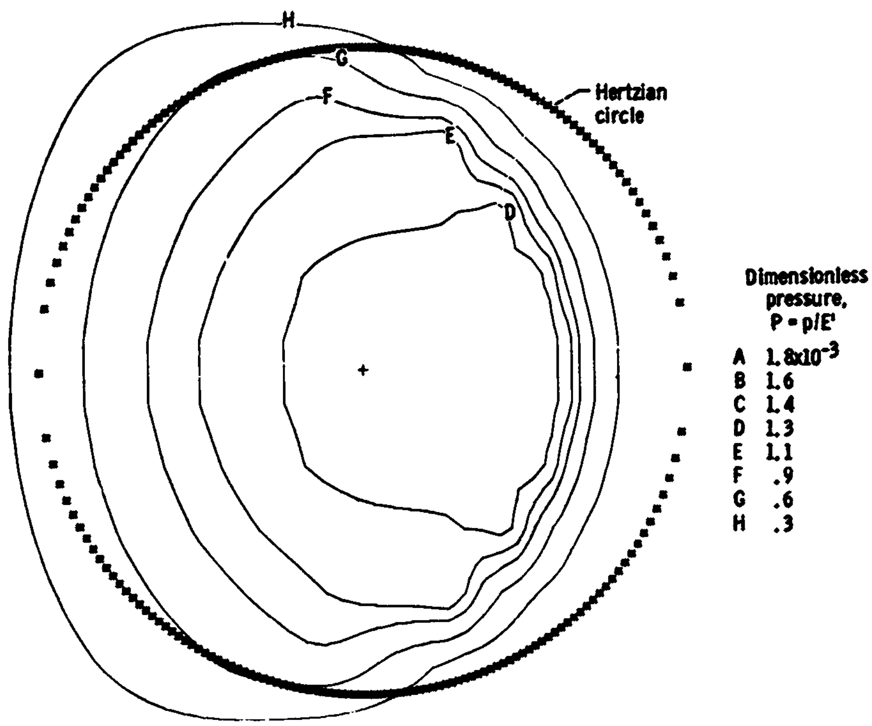
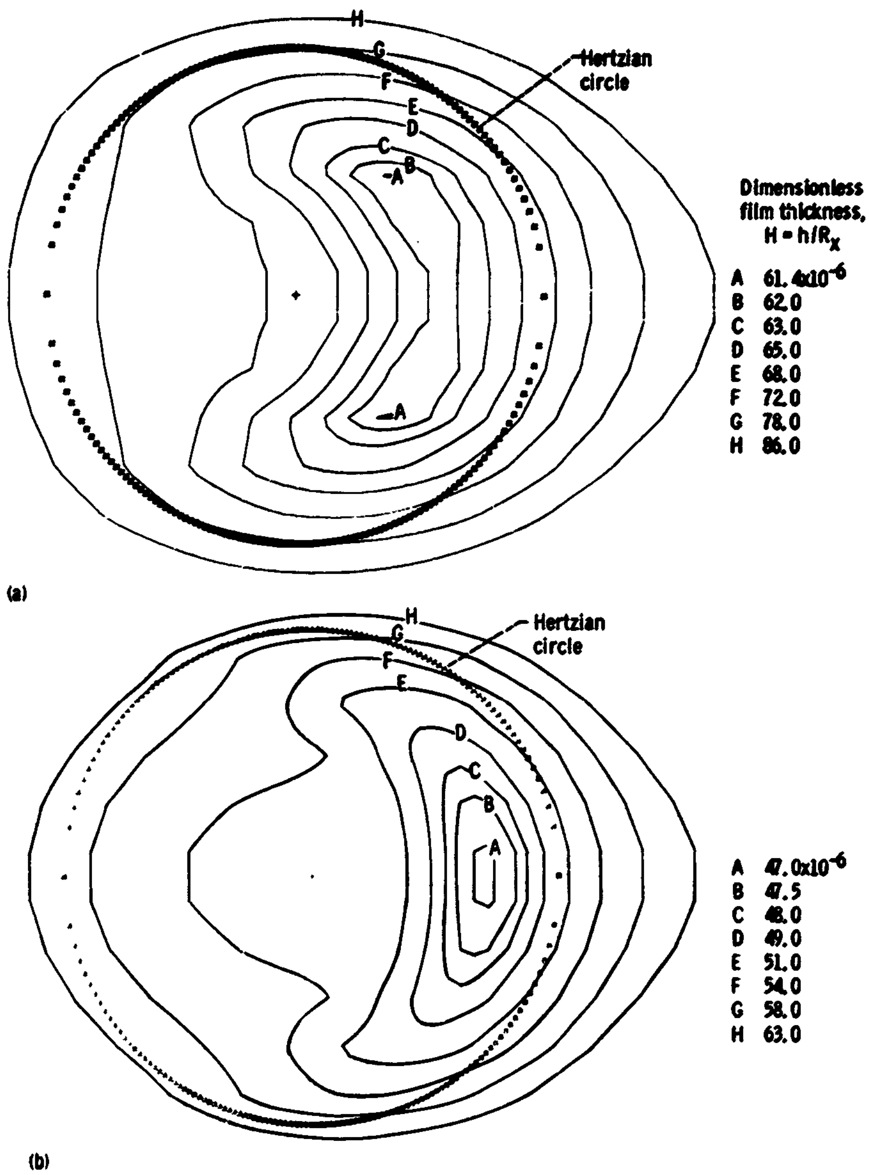


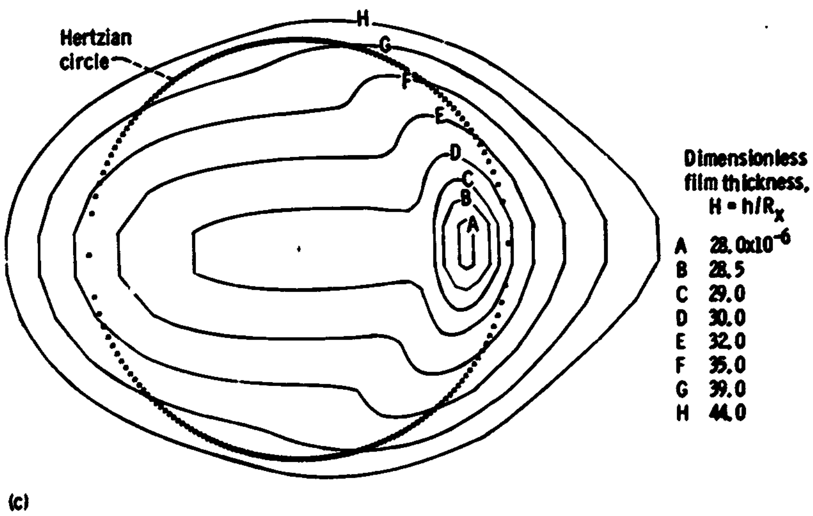
(c) $\tilde{m} = 1.5$.

Figure 9.5. - Concluded.



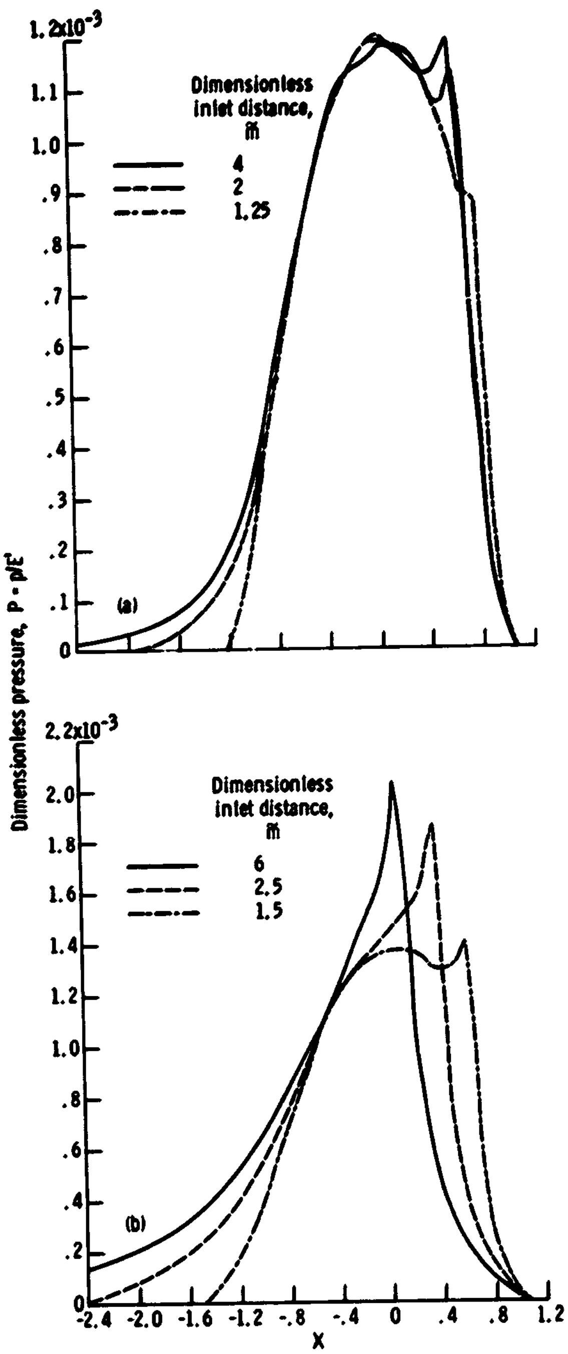
(a) $\tilde{m} = 6$.
(b) $\tilde{m} = 2.5$.

Figure 9.6. - Contour plots of dimensionless film thickness for dimensionless inlet distances $\tilde{m}$ of 6, 2.5, and 1.5 and for group 3 of Table 9.2.



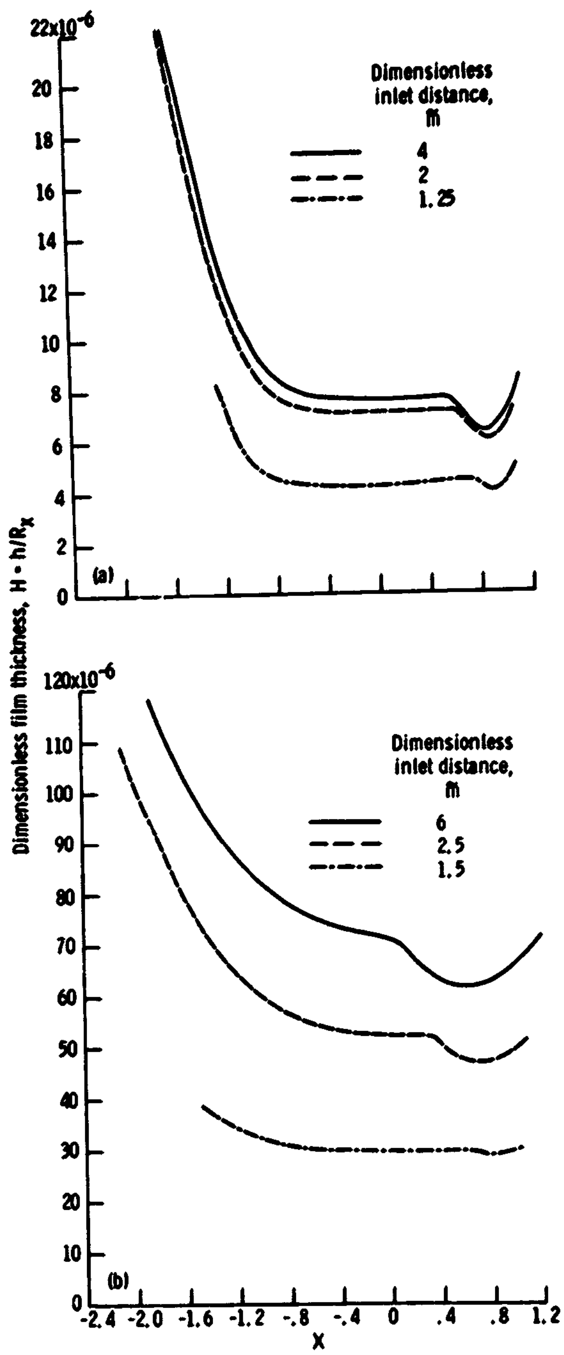
(c) $\tilde{m} = 1.5$.

Figure 9.6. - Concluded.



(a) Dimensionless parameters U and W held constant as given in group 1 of Table 9.2.
(b) Dimensionless parameters U and W held constant as given in group 3 of Table 9.2.

Figure 9.7. - Dimensionless pressure on X axis for three values of dimensionless inlet distance. The value of Y is held fixed near axial center of contact.



(a) Dimensionless parameters U and W held constant as given in group 1 of Table 9.2.
(b) Dimensionless parameters U and W held constant as given in group 3 of Table 9.2.

Figure 9.8. - Dimensionless film thickness on X axis for three values of dimensionless inlet distance. The value of Y is held fixed near axial center of contact.